Quiz 1.1: Sample Answers

For all these questions, we use the fact that the slope of the secant line between x_1 and x_2, and the average rate of change/velocity from x_1 to x_2 both have the same formula:

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

1. The displacement of an object moving along a straight line is given by $s = (2\sin(\pi t) + 2\cos(\pi t))$. Find the average velocity between 1 and 1.1 seconds.

   We use the formula to get average velocity:

   $$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{[2\sin(\pi 1.1) + 2\cos(\pi 1.1)] - [2\sin(\pi 1) + 2\cos(\pi 1)]}{0.1}$$

   which simplies to (be sure to have your calculator set to radians to deal with sin and cos):

   $$\frac{-2.520 + 2}{0.1} = -5.201.$$

2. Find the slope of the secant line between the point P with $x = 5$ and the point Q with $x = 5.1$, to the curve $f(x) = \frac{x}{1+3x}$.

   Again, we use the formula to get the slope of the secant line:

   $$\frac{f(5.1) - f(5)}{5.1 - 5} = \frac{\frac{5.1}{1+3(5.1)} - \frac{5}{1+3(5)}}{0.1} = \frac{0.3129 - 0.3125}{0.1} = 0.004$$

3. Find the slope of the secant line between the point P with $x = 3$ and the point Q with $x = 3.2$, to the curve $f(x) = \ln(5x)$.

   Again, we use the formula:

   $$\frac{f(3.2) - f(3)}{3.2 - 3} = \frac{\ln(5 * 3.2) - \ln(5 * 3)}{0.2} = 0.323$$

4. Find the average rate of change of $f(x) = 2x^2 + 3x$ from $x = 2$ to $x = 3$.

   We use the usual formula:

$$\frac{f(3) - f(2)}{3 - 2} = \frac{[2(3)^2 + 3(3)] - [2(2)^2 + 3(2)]}{1} = 13.$$

5. Find the average rate of change of $f(x) = 2x^3 + 2x^2 + 3x + 2$ from $x = 2$ to $x = 3$.

   We use the usual formula:

$$\frac{f(3) - f(2)}{3 - 2} = \frac{[2(3)^3 + 2(3)^2 + 3(3) + 2] - [2(2)^3 + 2(2)^2 + 3(2) + 2]}{1} = 51.$$

6. The displacement of an object moving along a straight line is given by $s = t^3/7 - t^2/7$. Find the average velocity between 1 and 1.1 seconds.

   We use the usual formula:

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{[(1.1)^3/7 - (1.1)^2/7] - [(1)^3/7 - (1)^2/7]}{0.1} = 0.1729.$$